$$\langle p|x\rangle = (2\pi\hbar)^{-1/2}\, e^{-\frac{i}{\hbar}px}$$

Representación de posición

1D $\quad \langle x|\hat{x}|\psi\rangle = x\langle x|\psi\rangle = x\,\psi(x)$

3D $\quad \langle \vec{r}|\hat{x}|\psi\rangle = x\langle \vec{r}|\psi\rangle = x\,\psi(\vec{r})$

1D $\quad \langle x|\hat{P}|\psi\rangle = -i\hbar \frac{d}{dx}\psi(x)$

3D $\quad \langle \vec{r}|\vec{P}|\psi\rangle = -i\hbar\nabla\langle \vec{r}|\psi\rangle$
$$= -i\hbar\nabla\psi(\vec{r})$$

Representación de momento

$$\langle p|\hat{P}|\psi\rangle = p\langle p|\psi\rangle = p\,\psi(p)$$

$$\langle p|\hat{x}|\psi\rangle = ??$$

El conmutador $[X, P]$

$$\langle\phi|[\hat{X},\hat{P}]|\psi\rangle = \langle\phi|\underset{\uparrow}{\,}[\hat{x},\hat{P}]|\psi\rangle$$

$$\mathbb{1} = \int dx\, |x\rangle\langle x|$$

$$= \int \phi^*(x)\langle x|\hat{x}\hat{P} - \hat{P}\hat{x}|\psi\rangle = \int \phi^*(x)\left[x\langle x|P|\psi\rangle + i\hbar\frac{\partial}{\partial x}(x\psi(x))\right]$$

$$= \int \phi^*(x)\left[-i\hbar x\frac{\partial}{\partial x}\psi(x) + i\hbar x\frac{\partial}{\partial x}\psi(x) + i\hbar\psi(x)\right]$$

$$= \int i\hbar\,\phi^*(x)\,\psi(x) = i\hbar\langle\phi|\psi\rangle$$

Como se vale $\forall\ |\phi\rangle, |\psi\rangle \Rightarrow [\hat{x},\hat{p}] = i\hbar$

Reglas de conmutación para $\vec{R}$ y $\vec{P}$.

$[R_i, R_j] = 0$, $[P_i, P_j] = 0$, $[R_i, P_j] = i\hbar\delta_{ij}$

Veremos que esto significa que no los podemos medir simultáneamente

4.2. Funciones de operadores

CT II.CBII.4

- $F(A) = \sum f_n A^n$
- (ejemplo) $e^A = \sum \frac{A^n}{n!}$
- Si $A\left|\phi_a\right\rangle = a\left|\phi_a\right\rangle$ entonces $A^n\left|\phi_a\right\rangle = a^n\left|\phi_a\right\rangle$
- $\therefore F(A)\left|\phi_a\right\rangle = \sum f_n a^n\left|\phi_a\right\rangle = F(a)\left|\phi_a\right\rangle$
- cuando $\left|\phi_a\right\rangle$ es un eV de A con ev a entonces $\left|\phi_a\right\rangle$ también es eV de $F(A)$ con ev $F(a)$.
- Ojo, hay que tener cuidado con $e^A e^B$, $e^B e^A$, e^{A+B}
- Si $[A, B] = 0$ si son iguales

WIKIPEDIA

Fórmula de Baker-Campbell-Hausdorff

Teorema relacionado con el álgebra de Lie

En matemáticas, la **fórmula de Baker-Campbell-Hausdorff** permite hallar la solución de Z para la ecuación

$$e^X e^Y = e^Z$$

con X e Y que pueden ser no conmutativos en el álgebra de Lie de un grupo de Lie. Hay varias formas de escribir la fórmula, pero todas finalmente producen una expresión para Z en términos algebraicos de Lie, es decir, como una serie formal (no necesariamente convergente) en X e Y y conmutadores iterados de los mismos. Los primeros términos de esta serie son:

$$Z = X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots,$$

Vamos a usar indistintamente $F(\hat{A})$ o $F(a)$
$\uparrow$ operador $\uparrow$ escalar

Formalmente $F(\hat{A}) : OP_H \to OP_H$ y $F(a) : \mathbb{C} \to \mathbb{C}$ son funciones distintas pero las usaremos indistintamente si los f_n en su serie son las mismas.

Reglas de cuantización:

¿Cómo construir operadores cuánticos?

Habíamos dicho $H_{\text{cuántico}}$ se obtiene de reemplazar cantidades físicas por operadores

With the position $\mathbf{r}(x, y, z)$ of the particle is associated the observable $\mathbf{R}(X, Y, Z)$. With the momentum $\mathbf{p}(p_x, p_y, p_z)$ of the particle is associated the observable $\mathbf{P}(P_x, P_y, P_z)$.

Recall that the components of $\mathbf{R}$ and $\mathbf{P}$ satisfy the canonical commutation relations [Chap. II, equations (E-30)]:

$$[R_i, R_j] = [P_i, P_j] = 0$$

$$[R_i, P_j] = i\hbar\,\delta_{ij} \tag{B-33}$$

A, one could simply replace[1], in the expression for $\mathcal{A}(\mathbf{r}, \mathbf{p}, t)$, the variables $\mathbf{r}$ and $\mathbf{p}$ by the observables $\mathbf{R}$ and $\mathbf{P}$:

$$A(t) = \mathcal{A}(\mathbf{R}, \mathbf{P}, t) \tag{B-34}$$

However, this mode of action would be, in general, ambiguous. Assume, for example, that in $\mathcal{A}(\mathbf{r}, \mathbf{p}, t)$ there appears a term of the form:

$$\mathbf{r} \cdot \mathbf{p} = xp_x + yp_y + zp_z \tag{B-35}$$

In classical mechanics, the scalar product $\mathbf{r} \cdot \mathbf{p}$ is commutative, and one can just as well write:

$$\mathbf{p} \cdot \mathbf{r} = p_x x + p_y y + p_z z \tag{B-36}$$

But when $\mathbf{r}$ and $\mathbf{p}$ are replaced by the corresponding observables $\mathbf{R}$ and $\mathbf{P}$, the operators obtained from (B-35) and (B-36) are not identical [see relations (B-33)]:

$$\mathbf{R} \cdot \mathbf{P} \neq \mathbf{P} \cdot \mathbf{R} \tag{B-37}$$

Moreover, neither $\mathbf{R} \cdot \mathbf{P}$ nor $\mathbf{P} \cdot \mathbf{R}$ is Hermitian:

$$(\mathbf{R} \cdot \mathbf{P})^\dagger = (XP_x + YP_y + ZP_z)^\dagger = \mathbf{P} \cdot \mathbf{R} \tag{B-38}$$

To the preceding postulates, therefore, must be added a symmetrization rule. For example, the observable associated with $\mathbf{r} \cdot \mathbf{p}$ will be:

$$\frac{1}{2}(\mathbf{R} \cdot \mathbf{P} + \mathbf{P} \cdot \mathbf{R}) \tag{B-39}$$

which is indeed Hermitian. For an observable which is more complicated than $\mathbf{R} \cdot \mathbf{P}$, an analogous symmetrization is to be performed.

The observable A which describes a classically defined physical quantity $\mathcal{A}$ is obtained by replacing, in the suitably symmetrized expression for $\mathcal{A}$, $\mathbf{r}$ and $\mathbf{p}$ by the observables $\mathbf{R}$ and $\mathbf{P}$ respectively.

We shall see, however, that there exist quantum physical quantities that have no classical equivalent and which are therefore defined directly by the corresponding observables (this is the case, for example, for particle spin).

4.5. Ec. de Schrödinger en $|\mathbf{r}\rangle$

CT II.CDII.c

- $i\hbar \frac{\mathrm{d}}{\mathrm{d}t} |\psi(t)\rangle = H\,|\psi(t)\rangle$
- $H = \frac{1}{2m} P^2 + V(R)$
- $$i\hbar \frac{\partial}{\partial t} \langle \mathbf{r}|\psi(t)\rangle = \frac{1}{2m} \langle r|P^2|\psi(t)\rangle + \langle r|V(R)|\psi(t)\rangle$$

En $\{|x\rangle\}$

Ya con esto $H = \frac{p^2}{2m} + V(x) \rightarrow \hat{H} = \frac{\hat{P}^2}{2m} + V(\hat{X})$ no depende de t

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle \xrightarrow{\text{en la rep } \{|x\rangle\}} i\hbar \frac{\partial}{\partial t}\langle x|\psi(t)\rangle = \frac{1}{2m}\langle x|P^2|\psi(t)\rangle + \langle x|V(\hat{X})|\psi(t)\rangle$$

$|\phi\rangle = P|\psi\rangle$

$$\langle x|P^2|\psi(t)\rangle = \langle x|P|\phi(t)\rangle = -i\hbar \frac{d}{dx}\phi(x,t) = (-i\hbar)^2 \frac{d^2}{dx^2}\psi(x,t)$$

$\langle x|\phi\rangle = \langle x|P|\psi\rangle$

$$\langle x|V(\hat{X})|\psi(t)\rangle = V(x)\,\psi(x,t)$$

explicar con calma esto usando su exp. en serie.

$$\boxed{i\hbar \frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\psi(x,t)}$$

En $\{|p\rangle\}$

CT D_{II}

Ec de Schrödinger en $\vec{p}$

$$i\hbar \frac{\partial}{\partial t}\langle \vec{p}|\psi(t)\rangle = \frac{1}{2m}\langle \vec{p}|P^2|\psi(t)\rangle + \langle \vec{p}|\vec{V}(\vec{R})|\psi(t)\rangle$$

$\int d^3p |p\rangle\langle p|$

$$i\hbar \frac{\partial}{\partial t}\bar{\psi}(\vec{p},t) = \left(\frac{p^2}{2m}\bar{\psi}(\vec{p},t) + \int \langle \vec{p}|V(\vec{R})|\vec{p}'\rangle\,\bar{\psi}(\vec{p}',t)\,d^3p'\right)$$

$\int d^3r |r\rangle\langle r|$ $\quad$ $\int d^3r' |r'\rangle\langle r'|$

$$\iiint \underbrace{\langle \vec{p}|\vec{r}\rangle}_{(2\pi\hbar)^{-3/2} e^{-\frac{i}{\hbar}\vec{p}\cdot\vec{r}}} \langle r|V(R)|r'\rangle \langle r'|p'\rangle\,\psi(\vec{p}',t)\,d^3p'\,d^3r\,d^3r'$$

$$(2\pi\hbar)^{-3}\int e^{-\frac{i}{\hbar}(\vec{p}\cdot\vec{r} - \vec{p}'\cdot\vec{r}')}\,V(r)\,\delta(\vec{r}-\vec{r}')\,\psi(\vec{p}',t)\,d^3p'\,d^3r\,d^3r'$$

transformada de Fourier

$$(2\pi\hbar)^{-3}\int e^{-\frac{i}{\hbar}(\vec{p}-\vec{p}')\cdot\vec{r}}\,V(r)\,d^3r\;\psi(\vec{p}',t)\,d^3p' = (2\pi\hbar)^{-3/2}\int \bar{V}(\vec{p}-\vec{p}')\,\psi(\vec{p}',t)\,d^3p'$$

$$\boxed{i\hbar \frac{\partial}{\partial t}\bar{\psi}(\vec{p},t) = \left(\frac{p^2}{2m}\bar{\psi}(\vec{p},t) + (2\pi\hbar)^{-3/2}\int \bar{V}(\vec{p}-\vec{p}')\,\psi(\vec{p}',t)\,d^3p'\right)}$$

Ec. de Schröd en la base de $\{|\vec{p}\rangle\}$.